# PIMS 2011 Workshop (Dr. CHIEN)

www.drchien.com carchien@yahoo.ca

## Factoring and Divisibility

Example 1: What is the LCM of the numbers from 1 to 10?

Example 2: 20! = 1x2x3x…x20. If the above is calculated, the answer will contain how many zeros at the end? For 30! ?

Example 3: a) How many divisors are there for the number 1000. b) if we were to multiply all the divisors together, what would the product be? c) if we were to add all the divisors together, what would the sum be?

## Fundamental Counting Principle:

When a result consists of separate parts, we can multiply these parts together to find the total number of ways the result can be obtained.

Example 4: A guy has 6 different colored shirts and 3 different pairs of pants. How many ways can he choose his outfit if he needs one shirt and one pant?

## Arrangement of n Objects:

The number of distinct arrangements of n objects can be expressed as n! = n(n-1)(n-2)…(3)(2)(1)

**Permutations** are the number of ways that n distinct things can be arranged if they things are taken r at a time. We write:
nPr = n(n-1)(n-2)…(n-r+1) = n!/(n-r)!
This is just a generalization of the fundamental counting principle when repetitions are not allowed.
Note that permutations apply only when repetitions are not allowed and when order is important.
Example 5: How many ways can we select 3 winners for $1^{st}$, $2^{nd}$, and $3^{rd}$ places for a race involving 12 runners?

**Combinations** are the number of ways that n distinct things can be arranged if the things are taken r at a time and the order of things is not important. We write:
nCr = (nPr)/r! = n!/(r!(n-r)!)

Combinations give the number of possible subsets.
when repetitions are not allowed and order is NOT important.

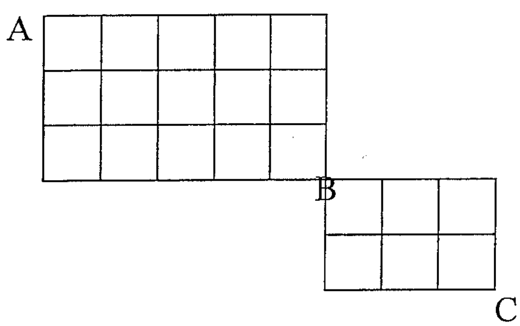
Example 6: How many ways can we select 3 volunteers from a group of 12 students?

Example 7: There are 8 empty seats on a bus and 5 people come on board. In how many ways can they be seated?

**Probability**

Probability is a measure of the chance that an event will occur.
P(event) = (number of desired outcomes)/(total number of possible outcomes)

Example 8: a) How many Paths are possible from point A to point B if all motion must be to the right or downwards? From point A to point C?
b) How many squares of all sizes are there?
c) How many rectangles?




Example 9: How many ways can three couples line up for a photo? What if each couple must stay together? What is the probability for that to happen?

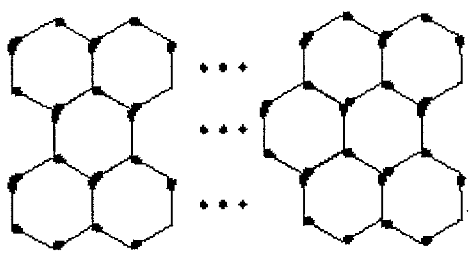
Example 10: Three married couples arrange themselves randomly in six consecutive seats in a row. Find the probability that the women will be in three adjacent seats.

Example 11: Mr. Smith keeps the phone numbers for his 10 closest friends (4 male and 6 female) in his digital phone memory. How many ways can he list them if no two male friends are listed next to each other? What is the probability for that to happen?

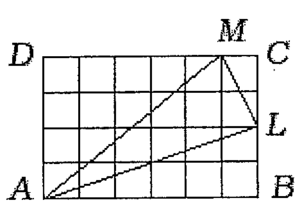
**More Problem Solving**

Example 12: The pattern consists of three rows of hexagons where the top and the bottom rows both have 20 hexagons and the middle row has 19 hexagons.

A single match is used to construct a side of each of the hexagons, and if two hexagons share a side, then a single match is used for that shared side. How many matches were used in total?



Example 13:The rectangle *ABCD* is made up of identical 1x1 squares. You choose a point at random inside rectangle *ABCD*. What is the probability that the point is inside triangle *ALM* ? What is the length of the largest side of triangle *ALM* ?



Example 14: Consider the set $\{a,b,c,d,e\}$. This set has five members. How many subsets of this set have either one, two, three, four, or five members?
Note: $\{b,e,d\}$ is the same 3-member subset of $\{a,b,c,d,e\}$ as $\{e,b,d\}$.

Example 15: Let $a\#b = a \times b + 2b$. What is the value of $(1\#2)\#3$?

Example 16: A 600 pound pumpkin has been entered in a contest. When it arrived, it was 99% water. After sitting in the warm sun for several days, it is 98.5% water. How much does it weigh now?

Example 17: A man is running through a train tunnel. When he is 2/5 of the way through, he hears a train that is approaching the tunnel from behind him at a sped of 60mph. Whether he runs ahead or back, he will reach an end of the tunnel at the same time the train reaches that end. At what rate, in miles per hour, is he running? (Assume he runs at a constant rate.)

Example 18: A soccer ball is sewn together from 32 pieces. Twelve of these are regular pentagons, and the rest are regular hexagons. The pentagons and hexagons all have the same side length. A seam has to be sewn wherever two pieces meat. How many seams are there?

Worksheet on Factoring and Divisibility

Example 1: What is the GCF of the numbers 63, 84, 294?

Example 2: What is the LCM of the numbers from 1 to 10?

Example 3: $20! = 1 \times 2 \times 3 \times 4 \ldots \times 20$ If the above is calculated, the answer will contain how many zeros at the end?

Example 4: For 30!, the answer will contain how many zeros at the end?

Example 5: How many factors of 2 does 20! have?

Example 6: How many divisors are there for the number 2000 and if we were to multiply all the divisors together, what would the product be?

Example 7: What is the smallest number with 36 factors?

Example 8: Suppose there are 1000 lockers and 1000 people. The first person opens all the lockers; the second person closes every second locker; the third person changes the state of every third locker (if the locker is open, he will close it, and if it is closed, he will open it); the fourth person changes the state of every fourth locker. This process continues, where the $n$th person changes the state of every $n$th locker. After all 1000 people have gone through, how many lockers are open?

Problem 1: How many natural numbers between 200 and 300 are divisible by 7?

Problem 2: Given that $3^n$ divides 15!, what is the greatest possible integral value of n?

Problem 3: Find the smallest positive integer divisible by 10, 11, and 12.

Problem 4: If the four-digit number 5, 7d2 is divisible by 18, what is d?

Problem 5: What is the smallest positive integer which has a remainder of one when divided by 2, 3, 4, 5, and 6?

Problem 6: Find the sum of all four-digit natural numbers which have a 4 in the thousands place and a 6 in the ones place, and which are divisible by 2, 3, 4, 6, 8, and 9.

Problem 7: What value can *a* have to make *a*74*a* divisible by 36?

Problem 8: What is the greatest value of n such that $2^n$ will evenly divide 16!

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Problem 9: How many factors of $2^{95}$ are there which are greater than 1,000,000?

Problem 10: Of the first 10,000 natural numbers, 5000 numbers have 2 as a factor; 3333 have 3 as a factor; 1000 have 10 as a factor; 666 have 15 as a factor; 333 have 30 as a factor. How many of the first 10,000 natural numbers have neither 2 nor 3 nor 5 as a factor?

Problem 11: What is the value of *a* in the following equation?

$$720 = 2^a \cdot 5 \cdot 3^2$$

Problem 12: How many factors of 21,600 are perfect squares?

Problem 13: How many factors does $2^3 \cdot 3^6 \cdot 5$ have?

Problem 14: What is the smallest positive integer *n* for which 88 is a factor of *n*! ?